

The University of Chicago

# STUDIES OF REFLECTION NEBULAE

A DISSERTATION SUBMITTED TO THE  
FACULTY OF THE DIVISION OF THE  
PHYSICAL SCIENCES IN CANDIDACY FOR  
THE DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF ASTRONOMY AND ASTROPHYSICS

1937

By

LOUIS GEORGE HENYEY

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## ON THE POLARIZATION OF LIGHT IN REFLECTION NEBULAE

L. G. HENYEV

### ABSTRACT

Observations for the purpose of detecting the presence of polarization in the light of the reflection nebula NGC 7023, the nebulae in the Pleiades and the nebulous patches associated with  $\gamma$  Cygni have been made through a Polaroid screen used as an analyzer. Each set of observations consisted of two plates having the principal plane of the Polaroid of one plate perpendicular to that of the other. The relative weakening or strengthening of the image along radii drawn from the star parallel to the positions of the principal plane of the Polaroid would indicate polarization.

Measurements on the plates of NGC 7023 indicate the presence of polarization up to 12 per cent. The plates of the Pleiades and of  $\gamma$  Cygni indicate little or no polarization.

These results are shown to be compatible with the reflection theory of diffuse nebulae, provided the reflecting particles are fairly large. In the case of the nebulae around  $\gamma$  Cygni, the lack of evidence of polarization does not favor the hypothesis of the presence of Rayleigh scattering in these objects. This hypothesis was suggested by the blue color of these nebulosities.

1. The investigation of polarization in reflection nebulae which is reported in the following article was undertaken to provide information concerning the nature of the nebular particles which scatter the light of the associated stars. Certain definite deductions are possible from a knowledge of the state of polarization, while certain other facts can be elucidated by combining the results of observations of polarization with results of other methods of attack.

The existence of polarization in any amount indicates that the scattering particles are not free atoms, for processes of emission all give rise to unpolarized light, except in the case of the scattering of light in a resonance line. But the observations of Meyer<sup>1</sup> on emission nebulae show that these objects exhibit no appreciable polarization.

If we know beforehand that a nebula is not of the emission type, observations of its polarization enable us to go a step farther. In general, if a particle scatters light in such a way that it does not hold the energy for any length of time but simply deflects it without change of wave-length, then the polarization of the deflected light has the following characteristics: (a) its plane of polarization is usually perpendicular to or, more rarely, parallel to the plane formed

<sup>1</sup> *Lick Obs. Bull.*, 10, 68, 1920.

by the incident and the scattered rays; and (*b*) the amount of polarization is inversely correlated with the size of the particles. When the scattering particles are of the order of size of a few hundred molecules or smaller we have what is usually known as Rayleigh scattering. Here the polarization reaches very large values, as is evidenced by our own blue atmosphere, which gives values up to 70 per cent. For larger particles the polarization is much smaller and is quite negligible, except for the case of specular reflection near the polarizing angle on the surfaces of very large bodies.

The orientation of the plane of polarization should give an independent confirmation of the fact that reflection nebulae are illuminated by stars. For if the nebula is sufficiently spread out, the plane of polarization will have different orientations at different points, depending on the direction of the line joining the star and the point in question. Consequently, if we observe polarization in a nebula such that its plane is symmetrical with respect to a star, we can safely say that this star is the source of illumination. It must be remembered that these relationships, strictly speaking, hold true only if we consider light which has suffered but one deflection after leaving the star; for light which has been deflected more than once the case is more complicated. However, in almost any case the planes of polarization of multiply scattered light will be so thoroughly mixed that all trace of polarization is lost. It therefore seems that radial polarization should be present.

The distribution of polarization over the apparent disk of a nebula could, under favorable circumstances, give additional information concerning the structure of the nebula. Here, unfortunately, the situation is much more difficult to analyze, since irregularities in the space density of the particles in the nebula can influence the result tremendously. But it may be well to keep this source of attack in mind, since undoubtedly in the future it will provide valuable information.

While several attempts have previously been made to detect traces of radial polarization in nebulae, no quantitative measures have been published. Most nebulae are faint and visual observations are therefore out of the question. Previous attempts to make observations, together with the difficulties encountered in the con-



struction of suitable analyzers, have been described by Dr. W. F. Meyer.<sup>2</sup> He found that the best results were obtained with a Nicol prism. The nebula was photographed in two perpendicular positions of the Nicol and the photographic densities of the two images were measured with a microphotometer. A comparison of the two sets of measures was then made to detect traces of polarization. Owing to the large optical thickness of the Nicol, it was necessary to collimate the light of the nebula before it passed through the prism. A camera lens on the other side of the Nicol focused the light on the photographic plate. With this arrangement Meyer obtained a series of observations of emission nebulae, of a number of spirals, and of one true reflection nebula, NGC 2261 (Hubble's Variable Nebula), in which we are here primarily interested. He found that in this object the polarization is small, probably less than 10 per cent. More recently Struve, Elvey, and Roach<sup>3</sup> found that the nebula near  $\rho$  Ophiuchi exhibits a definite but rather small amount of polarization, and they pointed out that this is compatible with the reflection theory. Since the effect is radial with respect to  $\rho$  Ophiuchi (unless the principal plane of the Polaroid was taken by chance in just the right direction), we have a confirmation of the association of the star and the nebula.

2. The detection of a state of radially symmetrical polarization can be very simply accomplished by the use of the Polaroid screen. Since the Polaroid is thin, it is unnecessary to collimate light which passes through it. The Polaroid is primarily a film of some celluloid-like material into which are imbedded numerous minute crystals of a double-refracting material. The optic axes of these crystals have been oriented into parallel directions, so that the film will polarize light which passes through it. This polarizing property is not complete and depends on the wave-length.<sup>4</sup> It is sufficient to state here that the region of transmission is toward the longer wave-lengths from 4200 Å, and that the region of good polarization (better than 80 per cent) is between this limit and about 7200 Å. The average polarization between these limits will depend on the color character-

<sup>2</sup> *Ibid.*

<sup>3</sup> *A. p. J.*, 84, 219, 1936.

<sup>4</sup> A description of the transmission and polarization of the Polaroid can be found in a paper by Ingersoll, Winans, and Krause, *J. Opt. Soc. Amer.*, 26, 233, 1936.

istics of the source and of the photographic plate, but it will usually be over 90 per cent. In the discussion which follows it will be assumed that the polarization of the Polaroid is 100 per cent.

On long exposures of bright stars, peculiar streaks were noticed passing through the images. Each streak is along a line parallel to the principal plane of the Polaroid, and decreases in intensity with increasing distance from the star, the total intensity depending on the density of the stellar image. The source of the effect is undoubtedly in the Polaroid and may be due to the scattering of the light in the polarized component removed from the principal beam. It was only on the plates of the Pleiades that the effect was noticeable, but even here little difficulty was experienced. It should be remarked that the effect is one which is in the opposite sense from the effect resulting from radial polarization.

The use of the Polaroid in detecting radial polarization is illustrated in Plate XXa. This plate shows two photographs, each taken through a Polaroid film, of a rectangular glass tank full of a colloidal dispersion of rosin in water which is illuminated by a light placed in the center. The center is greatly overexposed, so that the lamp is not visible. The lines represent the directions of the principal planes of the Polaroid in the two cases. Now the light scattered by such a medium is polarized in a plane perpendicular to the radius vector, so that the Polaroid will weaken the light to the greatest extent along the radii parallel to the principal planes. This is readily seen by comparing the two cases. A pair of such photographs of a nebula will then provide the means of detecting the presence of polarization.

3. In Figure 1 let  $O$  be a star illuminating a nebula surrounding it, and choose two points,  $P_1$  and  $P_2$ , lying on radii which are perpendicular to each other. Let the surface brightness at the two points be  $J_1$  and  $J_2$ , respectively, and the fraction of the light polarized be  $p_1$  and  $p_2$  in a plane perpendicular to the radius. Then the light intensity at each point can be resolved into two components, one along the radius and one along the tangent. The two components have intensities for  $P_1$

$$J_{1\text{Rad}} = \frac{1 - p_1}{2} J_1, \quad J_{1\text{Tan}} = \frac{1 + p_1}{2} J_1,$$



and for  $P_2$  corresponding values. Now if we take two photographs of such a nebula through a Polaroid disk such that the principal plane in one is along  $OP_1$  and in the other along  $OP_2$ , we will have

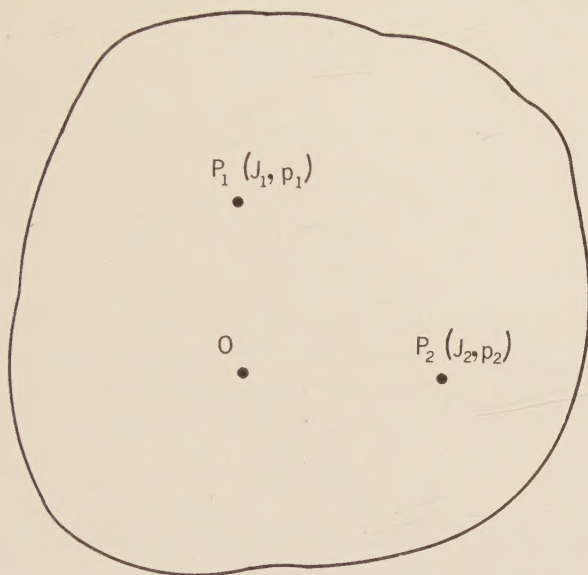


FIG. 1

but one of the components passing through. The transmitted light in each case is given below:

*Case A.*—Principal plane of Polaroid along  $OP_1$ :

$$J_{1A} = \frac{1 - p_1}{2} J_1, \quad J_{2A} = \frac{1 + p_2}{2} J_2.$$

*Case B.*—Principal plane of Polaroid along  $OP_2$ :

$$J_{1B} = \frac{1 + p_1}{2} J_1, \quad J_{2B} = \frac{1 - p_2}{2} J_2.$$

It is convenient to express surface brightnesses in terms of the sky as a unit. Now on different plates the sky will be somewhat different, so the intensities on different plates are not directly com-

parable. However, if we take the ratios of intensities on the two plates for both points, we have

$$\frac{\frac{J_{1A}}{J_{A, \text{sky}}}}{\frac{J_{1B}}{J_{B, \text{sky}}}} = \frac{1 - p_1}{1 + p_1} \cdot \frac{J_{B, \text{sky}}}{J_{A, \text{sky}}}, \quad \frac{\frac{J_{2A}}{J_{A, \text{sky}}}}{\frac{J_{2B}}{J_{B, \text{sky}}}} = \frac{1 + p_2}{1 - p_2} \cdot \frac{J_{B, \text{sky}}}{J_{A, \text{sky}}} \quad (1)$$

Taking the ratio of these two quantities, we have

$$\frac{\frac{J_{1A}}{J_{A, \text{sky}}}}{\frac{J_{1B}}{J_{B, \text{sky}}}} \div \frac{\frac{J_{2A}}{J_{A, \text{sky}}}}{\frac{J_{2B}}{J_{B, \text{sky}}}} = \frac{(1 - p_1)(1 - p_2)}{(1 + p_1)(1 + p_2)} \quad (2)$$

The left-hand side consists of known quantities, so we can compute the right-hand side. If we assume that the polarization is the same at the two points, we determine the quantity

$$\left( \frac{1 - p}{1 + p} \right)^2,$$

from which  $p$  can be computed. In any case, although  $p_1$  and  $p_2$  are different,  $p$  represents an average value.

4. The objects which have been examined for polarization are NGC 7023, the nebulae in the Pleiades and the nebulae around  $\gamma$  Cygni.

The plates of NGC 7023 have been obtained with the 24-inch reflector of the Yerkes Observatory, on Eastman Hyper-press plates. Each plate has been standardized with a tube sensitometer. The Polaroid used was a piece mounted in a bakelite rim having a clear aperture of 4 cm. It was mounted on the plateholder carrier of the reflector, a few millimeters in front of the plate, with the principal plane always parallel to the short dimension of the plate. Since the carrier may be rotated about the optical axis, the adjustment of the orientation could be made by rotating the carrier any amount desired, according to the divided circle on it. However, only two positions were used: the principal plane parallel and perpendicular to the hour circle through the nebula.

Two photographs are reproduced in Plate XXb. The straight

lines give the direction of the principal plane of the Polaroid. It is seen that an effect is present which is somewhat similar to the effect shown by the colloidal solution. The relative weakening of points along the two radii is, for points which show maximum effects, about 0.3 mag.

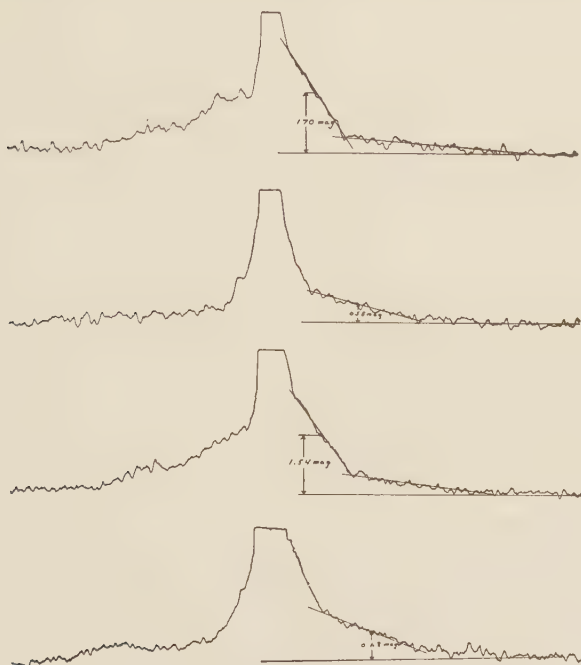


FIG. 2.—Microphotometer tracings of NGC 7023

The plates have been measured in two independent ways which give the same results. Microphotometer tracings have been made along the two radii. These are reproduced in Figure 2, where the process of reduction is indicated. The average density along two regions on the perpendicular radii was determined by drawing a straight line through the region and taking the density of the middle point on the line. The difference in magnitude between this point and the sky is shown in the figure. Measurements were also made at selected points along these radii by means of a Hartmann microphotometer. These measures are collected in Table I. The upper part of the table gives the results for one plate, the lower those for



a second plate. The principal plane of the Polaroid is indicated in the first column. The measures on the left are along the north-south radii, while those on the right are along the east-west radii. The table gives the measured difference in magnitude between the points on the nebula and on the sky, the corresponding intensity (plus that of the sky) in terms of the intensity of the sky, and the intensities corrected for the sky. The next to the bottom row gives the quantity involved in equation (1)—namely, the ratio of intensities of points on plate A to those on plate B. In the bottom row is the mean for the three sets of measures along each radius. Taking

TABLE I

| Plate                    |            | North-South Radii |       |      | East-West Radii |       |      |
|--------------------------|------------|-------------------|-------|------|-----------------|-------|------|
|                          |            |                   |       |      |                 |       |      |
| A. Polaroid NS . . . . . | — <i>m</i> | 1.54              | 1.41  | 1.62 | 0.68            | 0.58  | 0.79 |
|                          | I+1        | 4.13              | 3.66  | 4.45 | 1.87            | 1.71  | 2.07 |
|                          | I          | 3.13              | 2.66  | 3.45 | 0.87            | 0.71  | 1.07 |
| B. Polaroid EW . . . . . | — <i>m</i> | 1.70              | 1.47  | 1.80 | 0.55            | 0.42  | 0.65 |
|                          | I+1        | 4.79              | 3.87  | 5.25 | 1.66            | 1.47  | 1.82 |
|                          | I          | 3.79              | 2.87  | 4.25 | 0.66            | 0.47  | 0.82 |
| A/B. . . . .             | .....      | 0.85              | 0.93  | 0.81 | 1.32            | 1.51  | 1.30 |
| Mean. . . . .            | .....      | .....             | ..... | 0.86 | .....           | ..... | 1.38 |

the ratio of these values, we obtain for these points the mean of the quantity

$$\left( \frac{1+p}{1-p} \right)^2,$$

from which we find

$$p = 12 \text{ per cent.}$$

It should be repeated that this result is an average for those points which show large amounts of differences from plate to plate. Consequently, the average for the whole nebula is probably somewhat smaller.

The plates of the Pleiades were taken in the same manner as those of NGC 7023, except that Eastman Ortho-press plates were used. The exposures were approximately  $3\frac{1}{2}$  hours. The length of the ex-

posures and the brightness of some of the stars caused the appearance of the anomalous streaks already mentioned. However, the principal features of several nebulae were well marked, and comparisons could be made satisfactorily. The reader is referred to Plate 4 of Barnard's *Atlas of Selected Regions of the Milky Way*, on which the features described in the following discussion can be easily identified.

Surrounding Maia, there are a number of faint wisps of nebulosity and one very well-marked tonguelike patch curving from west of the star around the northwest and gradually fading away toward the north. Considerable weight can be given to an examination of this feature because of its large spread in position angle. No effect of polarization was found in it.

The results for Merope are, similarly, negative. Here too, there is a well-defined curved patch, running from the east to the north. No effect was noticed along the points of this nebulosity. A comparison of its eastern portion and of other patches nearer the star with the extended cloud south of the star also failed to reveal a relative change.

A small trace of polarization was found in the approximately straight filament running eastward from Electra. This filament was compared with another narrow and straight filament southeast of Electra and apparently running through a faint star (BD + 23° 510) from east to west. It was found that, when the Polaroid is parallel to the first filament, it is weakened with respect to the second. A comparison with the plate as a whole showed the same effect. It seems, therefore, that Electra provides the illumination for this nebula. The magnitude of the effect, even in this nebulous filament, is definitely less than that found in NGC 7023.

The plates of  $\gamma$  Cygni were taken with a Schmidt camera having a focal ratio of one to two. The camera has been described in the paper by Struve, Elvey, and Roach already referred to. The observing procedure is the same as that used by these observers. A visual examination of these plates reveals that the polarization, if present, is exceedingly weak. Owing to the small scale of the plates and the closeness of the star images, it did not seem advisable to measure these plates.

5. The results given in the foregoing section indicate that the polarization in these nebulae is quite weak. It is considerably weaker than the polarization of the corona,<sup>5</sup> for which the maximum is about 27 per cent. The polarization in NGC 7023 is about of the same order of magnitude as that found in the zodiacal light, namely, 15 per cent.<sup>6</sup> This small amount is in accordance with the reflection theory. Scattering by very small particles, such as Rayleigh scattering, would, for a probable distribution of the particles of the nebula in space, give values of the polarization much larger than this — perhaps between 30 and 70 per cent. The small amount of polarization in NGC 7023, in the nebulae of the Pleiades and around  $\rho$  Ophiuchi justifies the conclusion that the scattering particles are not of molecular size, or of a size comparable with, or smaller than, the wave-length of light. Since color observations of these nebulae have already shown<sup>7</sup> that they are not appreciably bluer than the illuminating stars, it is permissible to assume that the nebular particles are fairly large in size and that they scatter the light of the neighboring stars in accordance with the theory of Seeliger. A discussion of this theory will be given in another paper.

If Rayleigh scattering is indeed ruled out for the nebulosity associated with  $\gamma$  Cygni, then we should not expect selective scattering to take place. This point seems to be at variance with the color index for this object.<sup>8</sup> An explanation of this discrepancy may be found in the spectrum of the nebulosity. If there are any emission lines, they may give rise to such a spurious result. On the other hand, the presence of emission lines would not be expected from the spectrum of the exciting star, which is F8p, unless there are peculiar conditions of excitation. Further observations are needed before this point can be cleared up.

YERKES OBSERVATORY

October 1936

<sup>5</sup> Dufay and Grouiller (*C.R.*, **196**, 1574, 1933) find 26 per cent 10' from the solar limb, while Johnson (*Pub. A.S.P.*, **46**, 226, 1934) finds 28 per cent 8'.5 from the limb.

<sup>6</sup> Dufay, *C.R.*, **181**, 399, 1925.

<sup>7</sup> Struve, Elvey, and Keenan, *Ap. J.*, **77**, 274, 1933; Struve, Elvey, and Roach, *ibid.*, **84**, 219, 1936; and Keenan, *ibid.*, **84**, 600, 1936.

<sup>8</sup> Struve, Elvey, and Roach, *op. cit.*, p. 226.



# THE ILLUMINATION OF REFLECTION NEBULAE

L. G. HENYEV

## ABSTRACT

The general theory of the illumination of reflection nebulae by stars is discussed. It is shown that as far as large particles contribute to the scattering of the starlight, an equation of transfer, similar to the corresponding equation in the theory of the absorption and emission of light or in the theory of the scattering of light by small particles, is applicable. In this way it is possible to take into account the energy which has been scattered more than once by the particles of the nebula.

The theory has been translated into graphical form, by presenting a series of curves which give the distribution of the surface brightness on the faces of several ideal nebulae. Various important features of these curves are pointed out. When a nebula is illuminated by a star in front of it, the intensity gradient from the center outward varies inversely as the distance of the star from the nebula. When the star is inside of the medium, the gradient is even more steep. In the case of a thin nebula illuminated from behind, the bright nebula appears ring shaped if the particles in the nebula are large. For a more opaque nebula this ring shape is blurred over by the light which has suffered more than one reflection in passing through the medium.

The curves are applied to a number of actual nebulae. It is shown that the asymmetry of the nebulae IC 1284, IC 1287, and possibly BD+8°933 can be ascribed to an inclination of the faces of these nebulae to the line of sight. From the shape of the bright nebulosity near CD-24°12684 it is deduced that this star is immersed in the material. That this is the case is ascertained independently from the reddish color of the star and the nebula and from the faintness of the star, which seems to be cut down by about 3 mag. The nebula near  $\rho$  Ophiuchi is discussed and it is found that the star is about a parsec in front of the material.

## I

Investigations of diffuse nebulae have resolved themselves principally into two groups. One has been concerned with the primary source of the light-energy which comes to us from these objects. Owing to the efforts of a number of astronomers, today we feel confident that the source of this energy lies in stars which are in proximity to the nebulae and that the light from these stars either is reflected by the particles of the nebulae into space or is momentarily absorbed by atoms which eventually re-emit the energy. The other group of investigations has consisted of the collection of evidence concerning the actual process of the reflection or scattering of the star's light, and aimed thereby to discover the nature of the particles which do the scattering. It seems plausible, in the present state of our knowledge, to say that the largest part of the scattered light comes from particles of fairly large dimensions (large with respect to the wave-

length of light), and is produced by a process somewhat similar to diffuse reflection but with the possible presence of diffraction effects. Probably a small amount of light is scattered by small particles, selectively with respect to the color and to the plane of polarization of the incident light. We shall see that a certain amount of observational evidence indicated that these small particles may be numerous, but are inefficient scatterers as compared with their absorbing power. Studies of the distribution of light over the apparent disks of nebulae have been devoted to questions involved in one of these two groups; photometric observations for their own sake have been rare.<sup>1</sup> In this paper we shall be concerned with the principles involved in such an investigation, with the development of the general properties to be expected in an ideal reflection nebula, and with an application of these results, in a semi-qualitative manner, to the distribution of light over the apparent disks of several actual nebulae.

The theory of the illumination of a dust cloud by a point source has been formulated in considerable detail in a series of papers by Seeliger.<sup>2</sup> An account of these papers may be found in the article by Schoenberg in the *Handbuch der Astrophysik*.<sup>3</sup>

Seeliger considers the intensity of the light reflected by each particle in the direction of the observer. The amount of energy which we receive from such a particle depends on four factors: (1) the distance of the particle from the source; (2) the degree to which other particles weaken, by occultations, the light falling on our particle; (3) the phase angle under which the light is reflected; and (4) the weakening, by occultations, of the light coming to us from the particle. The surface brightness of the nebula at any point on its disk is then equal to the contribution, within unit solid angle, of all the particles in a column with unit cross-sectional area, and directed along the line of sight.

Seeliger does not consider the contribution to the energy which we receive by reflections of the light coming from other particles; that

<sup>1</sup> The work of Keenan (*Ap. J.*, **84**, 600, 1936) on NGC 7023 is the only one dealing with nebulae having continuous spectra.

<sup>2</sup> "Zur Theorie der Beleuchtung der grossen Planeten, etc.," *Abhandlungen der Bayer. Akad.*, **16**, 1887; "Theorie der Beleuchtung staubförmiger Massen, etc.," *ibid.*, **18**, 1893; "Ueber kosmische Staubmassen, etc.," *Sitzung. d. Akad. zu Muenchen*, **3**, 1901.

<sup>3</sup> **2**, 130 ff. and esp. 163 ff.

is to say, he neglects the energy which has suffered more than one reflection during the interval of time which elapses after it leaves the source and until it again escapes from the nebula. In certain cases the effect of multiple reflections is not negligible, and consequently in a complete general theory they must be considered. This is done in the next section. We assume that the scattered light in reflection nebulae comes from large particles. However, our general formulae remain correct for any type of scattering process, and it is only on page 119, where we introduce the Euler phase function, and again on page 121, where we begin to use the Lambert phase function, that the discussion becomes limited to the case of large particles. It would be relatively easy to use other phase functions (e.g., those derived by Blumer from the equations of Mie), but it seems best not to complicate the analysis and to carry through our computations for the Lambert phase function, which, of course, is correct only for large spherical particles having smooth surfaces. The success of Seeliger's work in explaining the rings of Saturn and the zodiacal light is sufficient justification for our procedure. While I have not succeeded in finding a nebula which would give a rigorous test of the phase function, the methods for such a test have been derived, and their application has been illustrated by means of a few examples.

## II

In the derivation of the fundamental equations governing the illumination of dust clouds we use what might be called a "microscopic" viewpoint as contrasted with the macroscopic viewpoint used by Seeliger. By this we mean that we consider the change which a ray of light undergoes within a small element of volume.

Consider a point and a ray, having a certain direction, situated in the nebula. Let us take an element of volume in the shape of a cylinder whose height is small compared with the diameter of its base. Let this cylinder be placed as shown in Figure 1, so that its base is perpendicular to the ray at the point. Represent its thickness by  $ds$  and the area of its base by  $d\sigma$ , the latter quantity being taken large compared to the cross-sectional area of the particles.

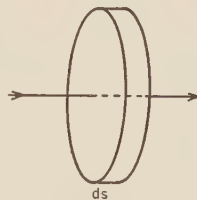


FIG. 1



The light coming in the direction of the ray and falling on the front surface of this element of volume is not uniformly distributed over the face. It may happen that particles, situated in front of the surface, are casting shadows on it, or that the same particles or others are reflecting light, coming from other directions, on it. Consequently, the illumination may vary from complete darkness to some maximum value. If the average value of the specific intensity over the surface is  $I$ , then the total energy falling on the face in unit time and in unit solid angle is given by

$$I d\sigma . \quad (1)$$

Before the light passes through the cylinder and falls on the back surface, part of its energy will be removed by falling on the particles in the volume. Take the average cross-sectional area of the particles to be  $\pi\rho^2$ , where  $\rho$  is the square root of the mean of the square of the radii. If the number of particles per unit volume is  $N$ , the total cross-sectional area exposed to the ray, in the element of volume, is

$$\pi\rho^2 N d\sigma ds .$$

Of course, it is possible that some of the particles are behind others and the total area is less than that stated, but if  $ds$  is chosen sufficiently small, the effect of such occultations over the distance  $ds$  is negligible. Not every fraction of this area is as effective as any other in absorbing light, since the illumination falling on  $d\sigma$  is not uniform. Since we are interested only in average values, it is permissible to take  $d\sigma$  sufficiently large so that, on the average, the irregularities cancel and we find that the light occulted by the particles is (per unit time and solid angle)

$$I \cdot \pi\rho^2 N d\sigma ds . \quad (2)$$

If the ray is inclined to the face of the cylinder, it is found that the amount of energy is given by the same formula.

In addition to the change in the intensity due to absorption, there is an increase due to the deflection of light from other directions into the one in which we are interested. We have seen that the

amount of energy absorbed from such an inclined ray is (per unit time and solid angle)

$$I' \cdot \pi \rho^2 N d\sigma ds ,$$

where  $I'$  is its average intensity. A certain fraction of this energy is deflected in the direction of the first ray. This fraction depends on the directions of the two rays and on the kind of particles involved. We will denote it by  $\Phi$ . Consequently, the energy deflected is

$$\pi \rho^2 N I' d\sigma ds \cdot \Phi .$$

The total contribution is this quantity summed over all directions, or

$$\pi \rho^2 N d\sigma ds \oint I' \Phi d\omega , \quad (3)$$

where the notation  $\oint d\omega$  represents the integral of the integrand over all solid angles.

Accordingly, the net gain in energy is

$$\pi \rho^2 N d\sigma ds \oint I' \Phi d\omega - \pi \rho^2 N I d\sigma ds . \quad (4)$$

If the specific intensity of the light falling on the back surface of our element of volume is  $I + dI$ , then, according to (1) and (4),

$$(I + dI)d\sigma - Id\sigma = \pi \rho^2 N d\sigma ds \oint I' \Phi d\omega - \pi \rho^2 N I d\sigma ds ,$$

or, after some reduction,

$$\frac{1}{k} \frac{dI}{ds} + I = \oint I' \Phi d\omega , \quad (5)$$

where  $k$ , the absorption coefficient, is written for  $\pi \rho^2 N$ .

It will be seen that equation (5) is entirely similar to the corresponding relationship in the theory of absorption and emission of light or in the theory of scattering by small particles. Suppose that scattering particles are present in addition to reflecting particles. We must then replace (5) by the following equation:

$$\frac{1}{k_1 + k_2} \frac{dI}{ds} + I = \oint I' \frac{k_1 \Phi_1 + k_2 \Phi_2}{k_1 + k_2} d\omega , \quad (6)$$

where the subscript 1 refers to the reflecting particles and the subscript 2 refers to the scattering particles. The total absorption coefficient is therefore the sum of the individual absorption coefficients and the net phase function is the weighted mean of the individual phase functions. This statement holds for any admixture of any number of different kinds of particles.

The quantity  $\Phi$  is, in the simplest case, a function only of the angle by which the light is deflected, or the phase angle, provided we refer to mean values. It is possible that, in some cases, it also depends on a second angle which determines the plane of the incident and of the reflected rays. What is most troublesome in this more general case is that the dependence of  $\Phi$  on this second angle is not an explicit one, but involves other circumstances. For instance, if the rays have a definite amount of polarization, and if the reflecting or diffusing surfaces are such that the reflectivity depends on the plane of polarization of a plane-polarized ray, the reflection in a certain direction is different for the two polarized components of a partially polarized ray. The two components must be taken such that one is in the plane of the incident and reflected rays, and the other is perpendicular to it. The function  $\Phi$  can then be conveniently replaced by two other functions,  $\Phi_1$  and  $\Phi_2$ , such that the first applies to one component and the latter to the other component. For mean values these functions depend only on the phase angle, except for preferentially oriented particles. It is necessary to specify additional quantities, defining the percentage of polarization and the position of the plane of maximum polarization, at each point in the medium and for every ray passing through the point.

Another property that the quantity  $\Phi$  may have is that of depending on the wave-length. Most simply, the dependence on the phase angle may be the same for all colors, so that  $\Phi$  is proportional to a certain function of this angle; in this case the constant of proportionality would depend on the wave-length. In any case one must investigate each color separately.

The dependence of the phase function on the phase angle may be extremely complicated, especially in the case of fairly large irregular

particles. When the diameters of the scattering particles are of the order of size of the wave-length of light, diffraction effects control, to a considerable extent, the nature of the phase function. Using the electromagnetic theory of light, Mie has investigated the process of scattering by small, spherical, and homogeneous particles, and among other important results has given various kinds of phase functions which depend on the size and the electromagnetic properties of the particles. For particles which are other than spherical the dependence is considerably different, but it is sufficiently well represented by the spherical case for qualitative purposes. An infinite variety of such functions is possible for different materials and sizes. This variety is lacking when one turns to larger particles: certain definite features are present when diffraction effects are negligible. The most important of these is the fact that no light is scattered forward in the direction of the incident light, since large opaque particles cast well-defined shadows in this direction. Although this is not a unique property of large particles, since in certain cases an approximately similar effect is present with small particles, the discovery of the effect in nebulae would be a strong point supporting the hypothesis of large particles. In a later paragraph I shall point out the conditions most favorable for observing this effect. At present definite evidence is not available for any known object in the sky.

With appropriate boundary conditions equation (5) serves in principle to determine the value of  $I$  at each point in space and for every direction. The boundary conditions consist of the values of  $I$  for each direction in space on a certain surface. Physically it is evident that this surface must be that at which a ray having a given direction enters the medium. This surface is then the outer boundary of the nebula, together with the surface of any stars which are imbedded in the nebula. The boundary conditions are consequently determined by the disposition of the external sources of illumination.

Formally, a solution of (5) is very easily obtainable. Let

$$I_0, I_1, I_2, \dots, I_i, \dots,$$



satisfy the equations

$$\left. \begin{aligned} \frac{1}{k} \frac{dI_0}{ds} + I_0 &= 0 \\ \frac{1}{k} \frac{dI_1}{ds} + I_1 &= \mathcal{F} I_0 \Phi d\omega \\ \cdot &\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \\ \frac{1}{k} \frac{dI_i}{ds} + I_i &= \mathcal{F} I_{i-1} \Phi d\omega \\ \cdot &\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \end{aligned} \right\} \quad (7)$$

Let the boundary values at the entrance surface be assigned to  $I_0$  and let the boundary values for the other  $I$ 's be equal to zero. Consequently, if we add all the equations in (7), we will have (5), provided

$$I = I_0 + I_1 + I_2 + \dots + I_i + \dots \quad (8)$$

It can be shown that the series (8) converges, if

$$k \geq 0 \text{ and if } \mathcal{F} \Phi d\omega \leq 1.$$

Since these conditions are satisfied, (8) gives the solution of (5), and the problem has been reduced to the solution of (7). Now let

$$d\tau = k ds,$$

and write (7i) in the form

$$\frac{d}{d\tau} (e^\tau I_i) = e^\tau \mathcal{F} I_{i-1} \Phi d\omega.$$

We shall call  $\tau$  the optical distance. We have, by integrating this equation from the entrance surface ( $\tau = 0$ ) to the point in question,

$$e^\tau I_i - I_{i0} = \int_0^\tau e^{\tau'} d\tau' \mathcal{F} I_{i-1} \Phi d\omega,$$

or, since  $I_{i_0} = 0$  when  $i > 0$ ,

$$I_i = \int_0^\tau e^{-(\tau-\tau')} d\tau' \mathcal{F} I_{i-1} \Phi d\omega. \quad (9)$$

It is seen from these relations that  $I_0$  represents the intensity of the light which has come directly from the sources and has passed through a portion of the medium. The quantity  $I_1$  represents the intensity of the light which has been reflected once by the particles after leaving the sources, and correspondingly  $I_i$  represents that which has suffered  $i$ -reflections.

In most cases the source of illumination is a star, and it is more convenient to use the total intensity instead of the specific intensity, since a star is practically a point source. Therefore  $I_0$  must be replaced by  $E$ , where

$$E = \frac{E_0}{r^2} e^{-\tau_0}, \quad (10)$$

where  $\tau_0$  is the optical distance from the entrance surface.

Using (10), we have for  $I_1$

$$I_1 = \int_0^\tau e^{-(\tau-\tau')-\tau_0} \frac{E_0}{r^2} \Phi(\tau') d\tau', \quad (11)$$

where the quantities  $\tau_0$ ,  $r$ , and  $\Phi$  must be expressed in terms of  $\tau$ . This equation is identical with the equation of Seeliger giving the intensity of the first-order reflected light.

The separation of  $I$  into component intensities may be the preferable method of solving certain problems, but in other cases the direct solution of the fundamental equation (5) may be more convenient.

### III

Seeliger has computed the surface brightness of a uniform nebula, which is limited by parallel planes and which is illuminated by a distant source. It is convenient to have the specific intensity for every point and direction in the medium.

In Figure 2 let  $i$  be the angle of incidence of the light coming from the source. Then

$$E = \frac{E_0}{r^2} e^{-kz' \sec i}$$

for a point which is at a distance  $z'$  from the surface facing the star. Since the light from the source is approximately parallel, we have for the intensity in the direction of the arrow

$$I_1 = \frac{E_0}{r^2} \Phi \int_0^\tau e^{-(\tau - \tau') - kz' \sec i} d\tau'.$$

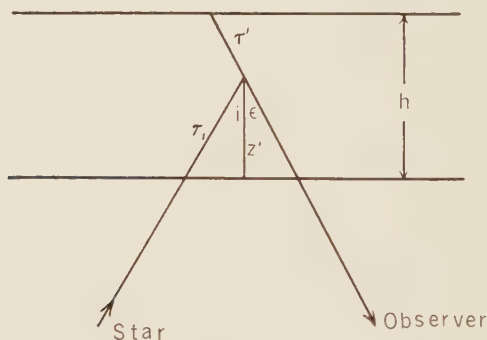


FIG. 2

Since the distance of the star from the nebula is large compared with the thickness of the nebula, the quantities  $r$  and  $\Phi$  may be considered constant. When

$$0 < \epsilon < \frac{\pi}{2},$$

$$\tau' = k(h - z') \sec \epsilon, \quad \tau = k(h - z) \sec \epsilon,$$

and

$$\begin{aligned} I_1 &= \frac{E_0}{r^2} \Phi (i + \epsilon) k \sec \epsilon \int_z^h e^{-k(z' - z) \sec \epsilon - kz' \sec i} dz' \\ &= \frac{E_0}{r^2} \Phi (i + \epsilon) \frac{\sec \epsilon}{\sec i + \sec \epsilon} e^{kz \sec \epsilon} (e^{-kz (\sec \epsilon + \sec i)} - e^{-kh (\sec \epsilon + \sec i)}). \quad (12) \end{aligned}$$

When

$$\frac{\pi}{2} < \epsilon < \pi,$$

$$\tau' = -kz' \sec \epsilon, \quad \tau = -kz \sec \epsilon$$

and

$$\begin{aligned} I_{\text{I}} &= -\frac{E_0}{r^2} \Phi(i + \epsilon) h \sec \epsilon \int_0^z e^{-k(z'-z) \sec \epsilon - kz' \sec i} dz' \\ &= -\frac{E_0}{r^2} \Phi(i + \epsilon) \frac{\sec \epsilon}{\sec i + \sec \epsilon} e^{kh \sec \epsilon} \{1 - e^{-kh(\sec \epsilon + \sec i)}\}. \end{aligned} \quad (13)$$

The surface brightness on the side facing the star is obtained by substituting  $z = 0$  in (12), that is,

$$I_{\text{I}} = \frac{E_0}{r^2} \Phi(i + \epsilon) \frac{\sec \epsilon}{\sec i + \sec \epsilon} \{1 - e^{-kh(\sec \epsilon + \sec i)}\}.$$

The surface brightness on the other side is found from (13) by substituting  $z = h$ :

$$I_{\text{I}} = -\frac{E_0}{r^2} \Phi(i + \epsilon) \frac{\sec \epsilon}{\sec i + \sec \epsilon} e^{kh \sec \epsilon} \{1 - e^{-kh(\sec \epsilon + \sec i)}\}.$$

As we have remarked, these relationships are valid when the source is sufficiently far from the nebula. This restriction need not be taken too seriously since the errors introduced by assumptions of uniformity may be more serious in actual applications.

In order to use these formulae we must be able to express  $r$  and  $i$  in terms of the apparent or angular distance of a point on the surface from the source. From Figure 3 we find that

$$\tan i = \frac{\delta}{x} \sec \epsilon - \tan \epsilon \quad \text{and} \quad r = x \sec i.$$

The results of equations (12) and (13) must be completed by the inclusion of the higher-order intensities. From the definition of these quantities we see that  $I_i$  is proportional to  $(\mathcal{F}\Phi d\omega)^i$ . Since this quantity, the albedo, is generally small, the higher-order intensities



approach zero. We will content ourselves with an examination of the order of size of  $I_2$ , for a plane parallel nebula illuminated by a distant source, and neglect all other terms.

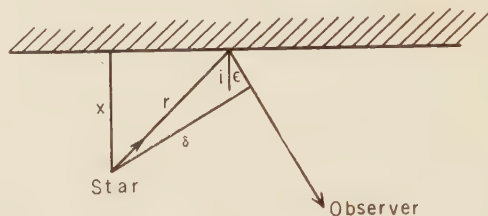


FIG. 3

From (9) we have

$$I_2 = \int_0^\tau e^{-(\tau-\tau')} d\tau' \oint I_1 \Phi d\omega.$$

As before, we take (using  $\phi$  as  $\epsilon$ , as in Fig. 2):

$$\tau' = k(h - z') \sec \phi, \quad \tau = k(h - z) \sec \phi,$$

when

$$0 < \phi < \frac{\pi}{2},$$

and

$$\tau' = -kz' \sec \phi, \quad \tau = -kz \sec \phi,$$

when

$$\frac{\pi}{2} < \phi < \pi.$$

The quantity  $I_1$  is given in equations (12) and (13), where  $z'$  must be substituted for  $z$  to avoid confusion. Let  $\theta'$  be the angle between the projection on the face of the nebula of the original incident ray coming from the source, and the projection of the direction of  $I_1$ . Also let  $\theta$  be the corresponding angle for  $I_2$ . Then

$$d\omega = \sin \epsilon \, d\epsilon \, d\theta'.$$

In the phase function we can no longer use  $i + \epsilon$ , since the third dimension is involved, and we introduce the angles  $\alpha'$  and  $\alpha$ , which

are, respectively, the supplements of the angles between  $I_0$  and  $I_1$ , and  $I_1$  and  $I_2$ . They are found from

$$\cos a' = \cos i \cos \epsilon - \sin i \sin \epsilon \cos \theta',$$

and

$$\cos \alpha = -\cos \epsilon \cos \varphi - \sin \epsilon \sin \varphi \cos (\theta - \theta').$$

The phase function we will use is that of Euler:<sup>4</sup>

$$\Phi(\alpha) = \frac{\gamma}{4\pi} (1 + \cos \alpha).$$

We have for the emergent light, when  $0 < \varphi < \pi/2$ ,

$$I_2 = k \sec \varphi \int_0^h \int_0^\pi \int_0^{2\pi} e^{-kz' \sec \varphi} \Phi(\alpha) I_1 \sin \epsilon \, d\theta' d\epsilon \, dz';$$

and when  $\pi/2 < \phi < \pi$ ,

$$I_2 = -k \sec \varphi \int_0^h \int_0^\pi \int_0^{2\pi} e^{k(h-z') \sec \varphi} \Phi(\alpha) I_1 \sin \epsilon \, d\theta' d\epsilon \, dz'.$$

To get an estimate of the order of magnitude of the effect we shall simply consider the case when  $i = 0$  and when  $\phi = 0$  or  $180^\circ$ . Then, integrating for  $\theta'$ ,

$$I_2 = 2\pi k e^{-kh} \int_0^h \int_0^\pi e^{kz' \sec \varphi} \Phi(\alpha) I_1 \sin \epsilon \, d\epsilon \, dz',$$

since the integrand is independent of  $\theta'$ . Now

$$I_1 = \frac{E_0}{r^2} \Phi(a') \frac{\sec \epsilon}{1 + \sec \epsilon} (e^{-kz'} - e^{-kh(1+\sec \epsilon)+kz' \sec \epsilon}),$$

when  $0 < \epsilon < \pi/2$ , and in the case when  $\pi/2 < \epsilon < \pi$ :

$$I_1 = -\frac{E_0}{r^2} \Phi(a') \frac{\sec \epsilon}{1 + \sec \epsilon} (e^{kz' \sec \epsilon} - e^{-kz'}).$$

<sup>4</sup> This phase function is adopted because of its simple form, and it bears a sufficient resemblance to the Lambert phase function to justify its use here. In general, the higher-order intensities are less sensitive to the shape of the phase function than is the first-order intensity.

Substituting and integrating for  $z'$ , we have

$$I_2 = 2\pi \frac{E_0}{r^2} e^{-kh} \left\{ \int_0^{\pi/2} \Phi(a)\Phi(a') \frac{\sin \epsilon}{1 + \cos \epsilon} \left[ kh - \frac{1 - e^{-kh(1+\sec \epsilon)}}{1 + \sec \epsilon} \right] d\epsilon \right. \\ \left. - \int_{\pi/2}^{\pi} \Phi(a)\Phi(a') \frac{\sin \epsilon}{1 + \cos \epsilon} \left[ \frac{e^{kh(1+\sec \epsilon)} - 1}{1 + \sec \epsilon} - kh \right] d\epsilon \right\} ;$$

or, if we put  $p = kh$  and substitute for  $\Phi$ :

$$I_2 = \frac{\gamma^2}{8\pi} \frac{E_0}{r^2} e^{-p} \left\{ \int_0^{\pi/2} (1 + \cos \epsilon) \sin \epsilon \left[ p - \frac{1 - e^{-p(1+\sec \epsilon)}}{1 + \sec \epsilon} \right] d\epsilon \right. \\ \left. - \int_{\pi/2}^{\pi} (1 + \cos \epsilon) \sin \epsilon \left[ \frac{e^{p(1+\sec \epsilon)} - 1}{1 + \sec \epsilon} - p \right] d\epsilon \right\} .$$

Now if we place  $u = \sec \epsilon$  in the first integral and  $u = -\sec \epsilon$  in the second one, we can reduce the expression to

$$I_2 = \frac{\gamma^2}{8\pi} \frac{E_0}{r^2} e^{-p} \left\{ \int_1^{\infty} \frac{1+u}{u^3} \left[ p - \frac{1 - e^{-p(1+u)}}{1+u} \right] du \right. \\ \left. + \int_1^{\infty} \frac{u-1}{u^3} \left[ p - \frac{1 - e^{-p(u-1)}}{u-1} \right] du \right\} .$$

Combining similar terms, we get

$$I_2 = \frac{\gamma^2}{8\pi} \frac{E_0}{r^2} e^{-p} \left\{ 2p \int_1^{\infty} \frac{du}{u^2} - 2 \int_1^{\infty} \frac{du}{u^3} + (e^{-p} + e^p) \int_1^{\infty} \frac{e^{-pu}}{u^3} du \right\} .$$

Also making use of the formula

$$\int_1^{\infty} \frac{e^{-qx}}{x^3} dx = \frac{1}{2} [e^{-q}(1-q) - q^2 Ei(-q)] ,$$

where  $Ei(q)$  is the exponential integral function

$$\int_{-q}^{\infty} \frac{e^{-x}}{x} dx ,$$

we arrive at the expression

$$I_2 = \frac{\gamma^2}{16\pi} \frac{E_0}{r^2} \left\{ e^{-p}(3p-1) - e^{-3p}(p-1) - p^2(e^{-2p}+1)Ei(-p) \right\} .$$

In a similar manner we can evaluate the case of  $\varphi = 0^\circ$ . For this the final result is

$$I_2 = \frac{\gamma^2}{8\pi} \frac{E_0}{r^2} \left\{ \frac{1 - e^{-2p}}{2} - pe^{-p} - p^2 e^{-p} Ei(-p) \right\}.$$

The values of the exponential integral function are available in the tables of Jahnke and Emde.<sup>5</sup>

The curves shown in Figure 4 represent the surface brightness of a thin nebula as a function of  $\delta/x$ . In other words, the distance  $x$ , between the star and the nebula, has been chosen as the unit of distance. The first set of curves has been computed for various values of  $\epsilon$  less than  $90^\circ$ , and the second for values greater than  $90^\circ$ . The angle  $\epsilon$  is the inclination of the nebula with respect to the sky, values larger than  $90^\circ$  indicating that the star is behind the nebula. In the first set the nebula is taken to be completely opaque, while in the second it is taken to have an optical thickness of 1 mag. By the optical thickness in magnitudes we mean the value of the quantity

$$2.5 \cdot \log_{10} e \cdot kh.$$

In addition, Figure 4 contains a set of curves for the surface brightnesses of nebulae having the various optical thicknesses shown accompanying each of the curves for the cases  $\epsilon = 0^\circ$  and  $\epsilon = 180^\circ$ . Only one half of each curve is given for  $\epsilon = 180^\circ$ ; the other half is symmetrical with the first.

In all the cases mentioned in the preceding paragraph the intensities are expressed in terms of the intensity of the starlight at unit distance (the distance  $x$ ), multiplied by the reduction factor, giving the absorption suffered by the starlight as observed from the earth. This reduction factor comes into consideration only when the star is behind the nebula. In all cases the intensities are given in magnitudes per square degree.

The Lambert phase function, which is proportional to

$$\frac{2}{3\pi^2} [\sin \theta + (\pi - \theta) \cos \theta],$$

<sup>5</sup> *Funktionentafeln*, Leipzig, 1933.



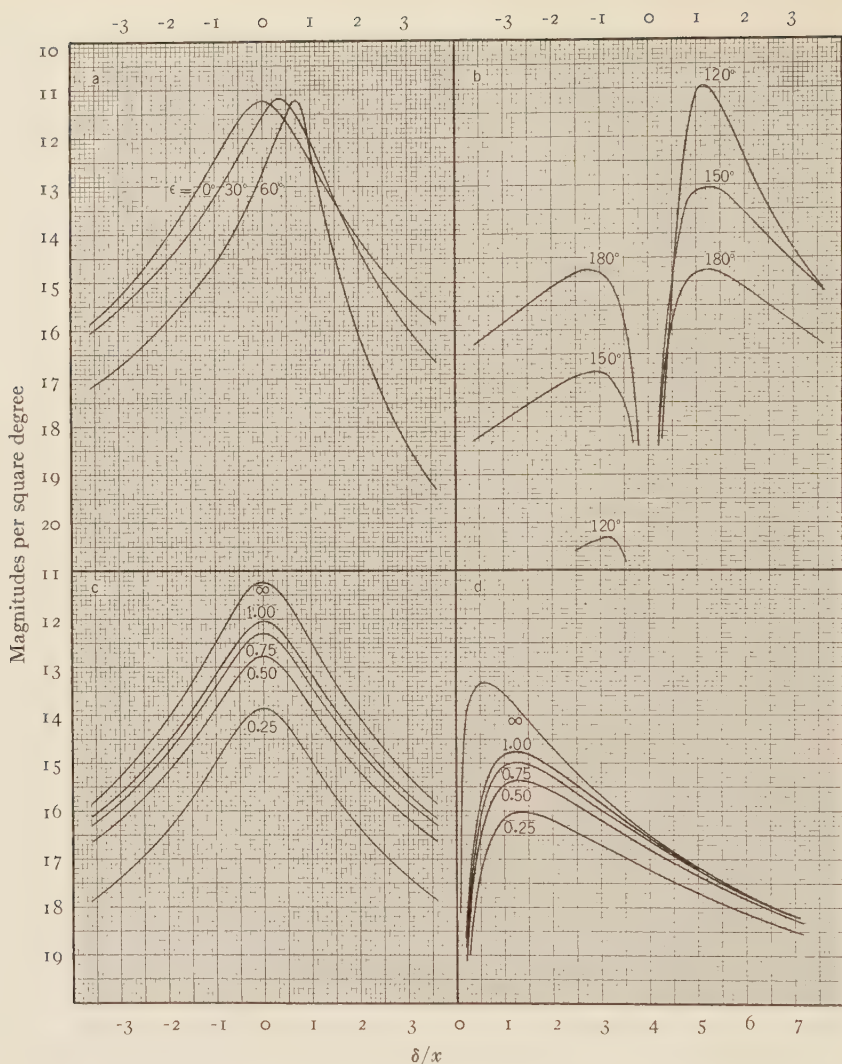


FIG. 4.—These curves display the distribution of brightness over a plane parallel nebula illuminated by a distant source. The albedo is taken to be unity. The Lambert phase function has been used. The unit of length is the perpendicular distance between star and nebula and the unit of intensity is the intensity of the star at this distance multiplied by the factor giving the obscuration of the star if the latter is seen through the nebula. The ordinates are given in magnitudes per square degree as referred to the unit, while the abscissa represents the quantity  $\delta/x$  (see Fig. 3). (a) An infinitely opaque nebula observed on the same side as the star with the face inclined to the plane of the sky at the angles shown. (b) A nebula 1 mag. thick observed on the opposite side from the star and inclined at the angles shown. (c) Similar to (a), but for inclination zero and for various opacities as given in magnitudes. (d) Similar to (b), but for inclination zero and for various opacities.

has been used in computing the curves in Figure 4. This function is applicable to large spherical objects whose surfaces scatter light in proportion to the products of the cosines of the angles of incidence and of reflection. Whether such a function is applicable to the particles in a nebula is, of course, uncertain. Since no observational material is available on the actual nature of the nebular phase function, we need not hesitate to use one which satisfies the general conditions imposed by the theory of reflection by large particles. As has been stated previously, the most characteristic feature of the phase function for large particles is the condition

$$\Phi(\pi) = 0.$$

Its agreement with this condition and its mathematical simplicity are the reasons for the use of the Lambert phase function in this section, and for the use of the Euler function in the section on second-order reflections.

To find the brightness for a certain albedo  $\gamma$ , where

$$\gamma = \oint \Phi d\omega,$$

we must multiply each intensity by this quantity.<sup>6</sup> This definition makes the albedo equal to the ratio of the total reflected light to the total incident light.

The surface brightness of a nebula which envelops its source was investigated by Seeliger, who derived an expression for the first-order intensity. The specific intensity in this case is

$$I_1 = \frac{2\gamma}{3\pi} \frac{E_0}{\delta^2} e^{\nu \cot a_0} \{ \psi_\nu(\pi - a_1) - \psi_\nu(\pi - a_0) \},$$

where  $\nu = k\delta$ ; the other quantities are explained in Figure 5. The function  $\psi_\nu(\theta)$  has been computed by Seeliger for the Lambert phase function and is tabulated in his third paper, to which we have referred. He has used  $\nu_1 = 0.4343\nu$  instead of  $\nu$ . Knowing the shape of the nebula and the position of the star, one can evaluate the angles

<sup>6</sup> This definition of the albedo is in many respects the most convenient one. Schoenberg uses another value which gives the specific intensity of the light reflected directly back by a single average particle.

$\alpha_0$  and  $\alpha_r$  in terms of  $\delta$ . These angles, together with the absorption coefficient, determine the values of  $\psi_r(\theta)$ .

The curves given in Figure 6a show the surface brightness as a function of  $\delta$ , for the case of a plane parallel and uniform nebula which has an optical thickness of one magnitude. The various cases shown represent the results for different distances of the star from the front surface of the nebula, the number accompanying each curve giving the ratio of this distance to the thickness of the medium. The unit of length is the distance between the two faces of the nebula, and the unit of intensity is the intensity of the starlight at unit distance multiplied by the exponential factor, giving the effect of

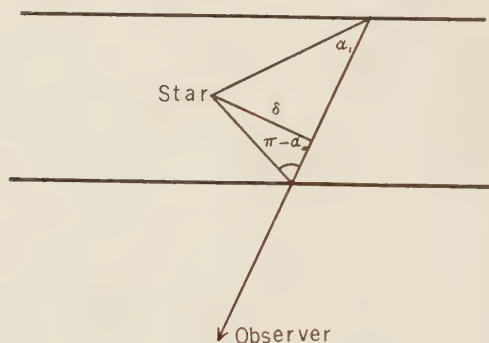


FIG. 5

the absorption along the line of sight. To reduce to a given albedo one must again multiply by  $\gamma$ . Figure 6b shows the same results in another form, the ordinates giving the intensities multiplied by the squares of the apparent distances,  $\delta$ , between the star and the nebular points considered. Hence they represent the deviation from an inverse square law of intensity variation.

A similar set of curves is given in Figure 6c. The nebula is taken to be infinitely opaque, and the star is placed at the different optical depths given by the numbers, in magnitudes, accompanying each curve. In these cases the unit of length is the actual distance of the star from the front surface, and the unit of intensity is the actual intensity of the starlight at this surface, including the effect of absorption.

We have now to examine the importance of higher-order reflec-

tions. I have computed  $I_2$  for two cases when the star is distant from the nebula: (a)  $\varphi = 0^\circ$  and (b)  $\varphi = 180^\circ$ , both for a plane parallel nebula and for  $i = 0^\circ$ . The results are shown in Figures 7a and 7b for various values of the optical thickness of the nebula.

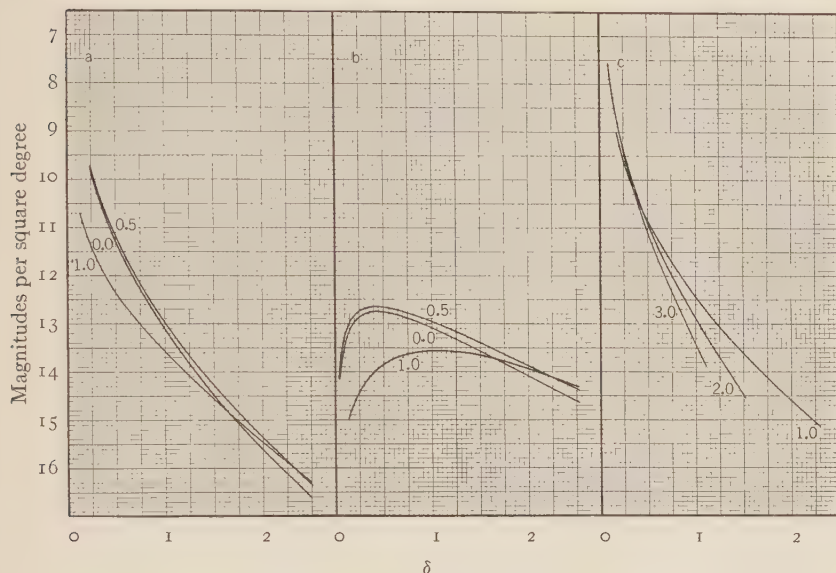


FIG. 6.—A plane parallel nebula internally illuminated. The unit of length is the distance between the faces and the unit of intensity is the star intensity at unit distance multiplied by the factor giving the absorption along the line of sight. The albedo is unity and the phase function which has been used is the Lambert. (a) The distribution for a nebula 1 mag. thick with the star placed at the various optical depths given in magnitudes. (b) Same as (a) but showing the deviation from an inverse square law of variation. (c) An infinitely opaque nebula with the star placed at the various optical depths shown. Here the unit of length is the actual distance of the star from the front face.

For an albedo  $\gamma$  the intensities must be multiplied by  $\gamma^2$ . The relative importance of the second-order effect compared with the first will be proportional to  $\gamma$ . When  $\epsilon = 0^\circ$  and  $p \rightarrow \infty$ , the maximum value of  $I_1$  will correspond to 11.2 mag. per square degree (from Fig. 4c). The corresponding intensity  $I_2$  will be 13.0 mag. per square degree. The difference in magnitude will give the ratio  $I_2/I_1$ , for a certain value of  $\gamma$ :

$$0.19\gamma.$$

For any reasonable value of  $\gamma$  ( $\frac{1}{3}$  or less) it will be seen that  $I_2$  may be neglected.

When the star is behind the nebula, the situation is entirely different. Using as the value of the albedo  $\frac{1}{5}$ , we find for a nebula having an optical thickness of 0.75 mag. that  $I_2$  corresponds to about 16.7 mag. per square degree. The maximum of  $I_1$  is for  $x$  equal to

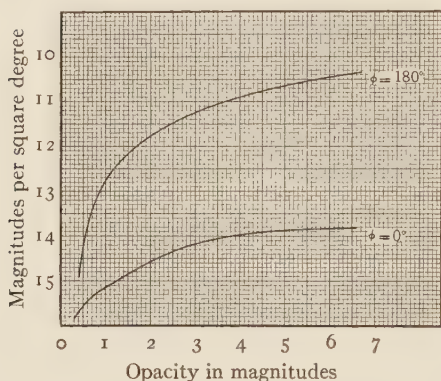


FIG. 7.—The second-order intensity for a plane parallel nebula. The albedo is taken to be unity. The phase function is the Euler. The units of length and intensity are the same as in Fig. 4. The intensity is shown for a point at the foot of the perpendicular from the star to the nebula and for the direction at right angles to the face. The intensity is shown for various opacities of the nebula.

$\phi = 0^\circ$ —the star in front of nebula  
 $\phi = 180^\circ$ —the star behind nebula

assumptions are: (1) the geometrical and physical uniformity of the nebula and (2) the applicability of the Lambert phase function.

Consider the case when the face of the nebula is perpendicular to the line of sight. When the star is in front of the nebula, the surface brightness near the projection of the star on the disk has a flat-topped maximum. If we now consider a position of the star nearer to the nebula, the surface brightness at each point increases. Since the curves in Figure 4 are drawn with the unit of length chosen equal to the distance between the star and the nebula, we shall see

about 1, and it corresponds to 16.8 mag. per square degree. The two intensities are nearly the same, while for denser nebulae  $I_2$  increases rapidly with increasing optical thickness, and becomes much more important than  $I_1$ . While no computations have been made for the third-order intensity, it is probable that this, too, would be large.

#### IV

The curves in Figures 4–6 display a number of features which, though depending on the simplifying assumptions we have made, are applicable in a qualitative way to actual nebulae. These as-



that, as the star approaches the nebula, these curves must be compressed in the direction of the abscissae. It will be seen that not only does the surface brightness increase but the curve representing it becomes steeper. For a position of the star on the front surface (see Fig. 6) the surface brightness is very great near the star and decreases very rapidly with increasing distance. Near the star the intensity varies as  $1/\delta$ .

For positions of the star farther into the nebula the distribution of intensity is similar, as we see from Figure 6. As the star approaches the back surface of the nebula, the distribution of the surface intensity near the star becomes gradually steeper, but when it assumes a position just outside the nebula the central intensity has a minimum value. Higher-order reflections will tend to fill in this minimum and will somewhat modify the conclusions. The surface intensity increases with increasing distance from the star and, after reaching a maximum, decreases again (see Fig. 4). For positions of the star farther away, the central minimum is broader and the maxima are farther apart. If the second-order intensity is extremely strong, no minimum may be observed.

It must be emphasized that the small central intensity which we predict for thin nebulae depends to a certain extent on the phase function we have chosen. We have assumed that the type of scattering which is most common is that of diffuse reflection by large particles, for which

$$\Phi(\pi) = 0.$$

There are processes of scattering which do not satisfy this condition, but they involve particles which are of the same order of size as, or smaller than, the wave-length of light. As has been pointed out previously, observational evidence definitely favors the view that the particles are large. However, in all the cases considered, it is simple to replace the Lambert phase function by any appropriate phase function in order to satisfy the conditions of a problem. In general, very little change will be produced in the shape of any of the curves, except Figure 4*d*, by such a substitution, unless the new phase function is such that  $\Phi(0)$  approaches zero. This is the case when the scattering particles are extremely small and scatter the light for-

ward instead of reflecting it back as in the case of large particles. Even then the intensity distribution over a nebula which is internally illuminated will not be affected greatly. But if the stellar source is outside of the medium, the situation is different. In the more important case of the star situated in front of the material, the first-order intensity may no longer be the largest part of the total intensity. It is obvious that a different treatment of the problem is needed. This problem is at present being investigated by the writer and will be discussed in a new paper.

An interesting feature of the curves is the asymmetry of the intensity distribution when a nebula is observed at an angle. The most important point which presents itself is the shift of the maximum surface brightness, depending on the inclination, when the star is in front of the nebula. These general features are independent of the phase function adopted. They are a result of the intensity distribution of the starlight falling on the nebula.

## V

In this section we shall apply the curves, which have been given above, to some actual nebulae. In most cases no accurate photometric data are available, so that our conclusions will be of a preliminary character. Even when accurate observations are available, care must be exercised in applying the theory, since structural irregularities always distort and mask the general features which have been discussed. Actual galactic nebulae are certainly not the ideal objects which we have theoretically considered.

With accurate observations a great deal of information may be derived. In the first place, it is possible to ascertain whether the source is or is not within the boundaries of the nebular material. If it is inside or on the farther side, a knowledge of its distance, luminosity, and apparent brightness will determine the optical thickness of the material in front of it.

Furthermore, when the star is in the foreground, its distance from the front surface of the nebula may be determined in terms of the extent of the nebula or, if the parallax is known, in absolute units. This quantity may be found with reasonable certainty, since the shape of the intensity profile as given in any of the sets of curves

(Fig. 4) is remarkably similar for varying thicknesses of the medium. The procedure is simply to fit the scale along the abscissa to the observed case, and, since the unit of length is the quantity sought, it will be determined. If the star is immersed in a very opaque nebula, one can, if the obscuration of the star is known, get its true distance from the front surface in an analogous manner. When the star is in a thin layer of material, the true thickness of the layer may be determined from the optical thickness. In these two cases the absorption coefficient is given by the ratio of the optical distance to the true distance.

When a star is in front of a nebula with its surface inclined to the line of sight, an estimate can be made of this inclination. This may best be done by first considering the intensity distribution along a line perpendicular to the axis of symmetry and passing through the star. We can determine from this distribution the distance, in relative or absolute units, between the star and the point on the nebula immediately behind it. This distance is

$$x \csc \epsilon .$$

The distance to the point of maximum intensity is, in the same units, approximately

$$x \sin \epsilon .$$

From these two numbers  $\epsilon$  may be computed.

One more important quantity is derivable from photometric data, namely, the albedo. Once the scale of the object has been determined, it is possible, in most cases, to derive this quantity from the relative brightness of the star and the nebula.

## VI

Plates I and II reproduce photographs of reflection nebulae, which I shall discuss in detail. These are copied from plates taken by Professor E. E. Barnard and used in his *Atlas of Selected Regions of the Milky Way*.

The nebula IC 1287 falls in a region of fairly uniform dark nebulosity. It can be seen that the nebulosity is, nevertheless, one-sided, and the maximum intensity seems to be southwest of the star. The

nebula IC 1284 shows a similar characteristic; it extends farther toward the south in a field of uniform dark material. It seems highly probable that the face of the nebula in each case is inclined slightly to the line of sight and produces the observed effect. The stars would have to lie in front of the material, and this seems reasonable since the intensity gradient is, in both cases, very flat.

Indeed, we should expect to observe the majority of nebulae at an angle, since the probability of occurrence of a given  $\epsilon$  is  $\sin \epsilon$ . The fact that most reflection nebulae are nearly or approximately symmetrical indicates that the stars illuminating the matter are generally near or actually inside of the nebula.

I have measured roughly the distance from the center of the nebula IC 1287 to the star and have found that  $x \sin \epsilon$  is about  $200''$ . The width of the nebula perpendicular to the line joining the center and the star is around  $750''$ . Let us assume, for the sake of illustrating the method of analysis, that this distance is roughly  $1.5x \csc \epsilon$ . Then

$$\begin{aligned} x \sin \epsilon &= 200'' , \\ x \csc \epsilon &= 500'' . \end{aligned}$$

We get from these relationships, very roughly, that

$$\epsilon = 35^\circ$$

and

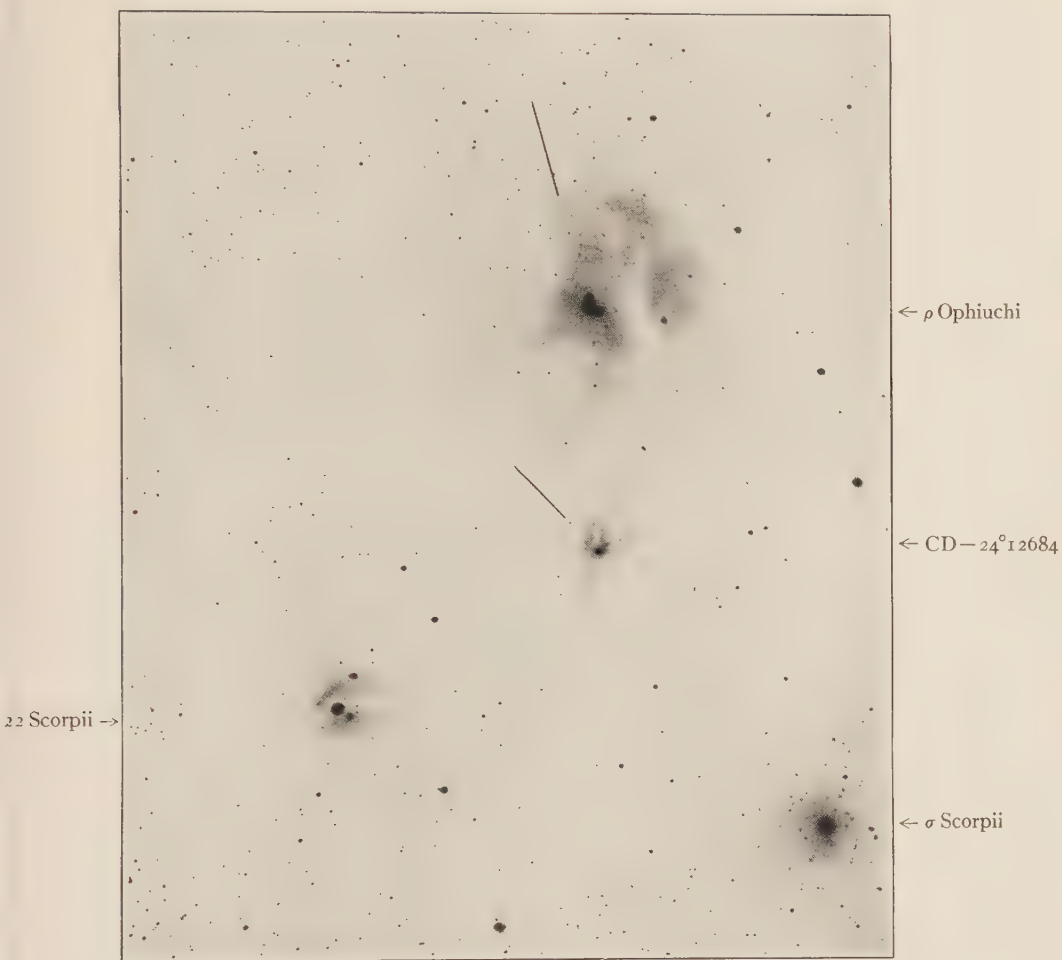
$$x = 300'' .$$

From the spectral type, B<sub>3</sub>, of BD-10°4713 I have estimated the distance of the star and nebula to be about 300 parsec. This gives

$$x = 100,000 \text{ astronomical units} .$$

Taking the absolute magnitude of a B<sub>3</sub> star as  $-1.0$  mag., the apparent magnitude of the star at the nebula may be estimated at  $-7.5$ . With an albedo of about  $\frac{1}{4}$ , the curve for  $30^\circ$  in Figure 4 gives as the surface brightness of the center of the nebula 5 mag. per square degree, which is half a magnitude fainter than the sky. These derived quantities are admittedly very rough, but their reasonableness is unquestionable and leads to the conclusion that the assumptions are consistent.

PLATE I

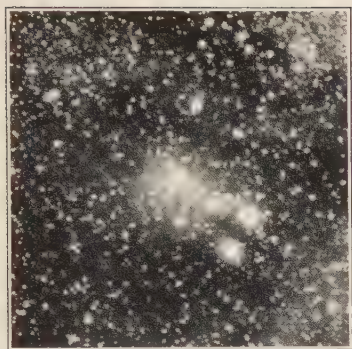


NEBULAE IN SCORPIUS AND OPHIUCHUS

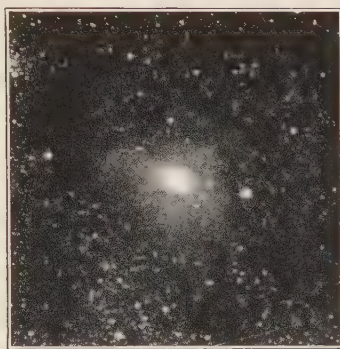




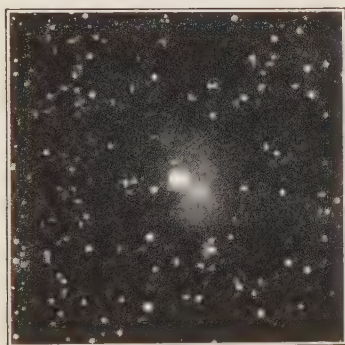
PLATE II



IC 1284



IC 1287



BD+8°933



The nebula around BD+8°933 is completely one-sided. There is dark material all around the star which is apparently connected with the bright patch. Since the material is almost completely dark next to the star, the latter cannot be immersed in it. It seems, then, that the nebula apparently near the star is in reality farther away, and we have possibly an extreme case of inclination. According to the curves in Figure 4*a* we should expect the intensity gradient to be smaller on the side of the maximum near the star. Since this is not the case, the distribution of the material must play a very important part in producing the fanlike shape of the bright nebula.

The group of nebulae in Scorpius and Ophiuchus, which are related to the extensive dark cloud in this region, contains some of the most interesting of these objects in the sky. The nebula which surrounds 22 Scorpii does so in a most peculiar manner. At first glance one is tempted to consider this system as a case of a star shining through a layer of nebulous matter. Indeed, some of the evidence points strongly toward such a hypothesis. The obscuring matter, which elsewhere is both dark and illuminated, extends right up to the star, and it is obvious that it is a part of the same cloud. This material, though not so dense as it is farther east, where it is practically opaque, seems to be fairly continuous. If it is behind the star, we should expect it to be uniformly illuminated. Very probably the particles are fairly large, since no evidence of selective scattering was found around  $\rho$  Ophiuchi and  $\alpha$  Scorpii and no evidence of strong polarization was found around the former.<sup>7</sup> The nebulae around these stars are connected continuously with one another and with that around 22 Scorpii. With large particles it is unlikely that the phase function is small for small phase angles. The lack of illumination near the star indicates that this matter must be in front of it. Since it is evident that very little second-order intensity is present, the albedo of the particles must be very small.

If we assume that this is indeed the case, we can estimate a lower limit for the absorption of the cloud. The distance from the bright ring to the star is about 250". Taking the distance of the system to be 100 parsec, the radius of the ring is 25,000 astronomical units. According to the curves for  $\epsilon = 180^\circ$ , this distance is slightly larger than the distance between the star and the nebula. We can say then

<sup>7</sup> Struve, Elvey, and Roach, *Ap. J.*, **84**, 219, 1936.

that this latter distance is about 20,000 astronomical units, which gives as the apparent magnitude of 22 Scorpii, as seen from the nebula (taking  $m = 4.9$ ),

$$4.9 - 5 \log \frac{100 \times 200,000}{20,000} = -10.1 \text{ mag.}$$

The maximum brightness of the nebula plus the sky is around 0.6 mag. brighter than the sky, making the nebula alone about 0.3 mag. fainter. Taking the brightness of the sky as 4.5 mag. per square degree, we have as the brightness of the nebula 4.8 mag. per square degree. For a given albedo we may find from these quantities the difference between star and nebula, or

$$4.8 + 2.5 \log \gamma + 10.1 = 14.9 + 2.5 \log \gamma.$$

Now, the albedo is, as we have said, very small. According to Figure 4*d*, we must then assume the nebula to be very opaque. For example, if  $\gamma = 0.1$  (which is unreasonably large in view of the absence of the second-order intensity), the difference between star and

TABLE 1

| STAR                  | SPECTRUM |        | MAG. | $M$  | $m-M$ | SEARES AND HUBBLE |      | INT. C.E. |
|-----------------------|----------|--------|------|------|-------|-------------------|------|-----------|
|                       | HD       | Mt. W. |      |      |       | Sp.               | C.E. |           |
| $\beta$ Scorpii.....  | B1       | Bon    | 2.90 | -3.1 | 6.0   |                   |      |           |
| $\sigma$ Scorpii..... | B1       | .....  | 3.08 | 2.4  | 5.5   | B1                | +1.2 | +0.5      |
| $\nu$ Scorpii.....    | B3       | B2n    | 4.29 | 1.5  | 5.8   |                   |      |           |
| CD-24°12684..         | B3       | .....  | 8.0  | 0.9  | 8.9   | B5                | 1.8  | 0.7       |
| 22 Scorpii.....       | B3       | B3n    | 4.87 | 0.9  | 5.8   | B5                | 0.2  | 0.1       |
| $\rho$ Ophiuchi.....  | B5       | B4n    | 5.22 | -0.6 | 5.8   | B5                | +1.1 | +0.4      |

nebula becomes 12.4 mag. This is considerably above the maximum value of the ordinates in Figure 4*d*, corresponding to a completely opaque nebula. However, the star 22 Scorpii, which would necessarily suffer considerable obscuration according to these assumptions, cannot be cut down more than a few tenths of a magnitude. We see this from Table 1, in which are tabulated a number of stars, includ-



ing 22 Scorpii, all lying at approximately the same distance from us. Also tabulated are the spectra, the apparent magnitudes, the mean absolute magnitudes corresponding to the given spectrum,<sup>8</sup> and the distance moduli. The last three columns give the spectral type and color excess according to Seares and Hubble<sup>9</sup> and their color excess reduced to the International System. These distance moduli should be the same for each of the stars unless they are affected by absorption. Except for one star, to which we shall refer later, they are indeed the same.

It appears, then, that on the basis of our assumption concerning 22 Scorpii and its companion nebula, and on the basis of the principles discussed in this paper, we are led to a contradiction in the interpretation of the facts. It appears quite certain that 22 Scorpii is in front of its nebula.

Another object of considerable interest is the nebulous star CD - 24° 12684. The rapid decrease in the surface brightness of its nebula with increasing distance from the star suggests that we have here an example of a star imbedded in the material which it illuminates. To demonstrate this, density measurements on the original Bruce plates of Barnard were made along the lines shown in Plate II, for this object and for comparison for the nebula associated with  $\rho$  Ophiuchi. These are shown, as galvanometer deflections, in Figure 8, and they demonstrate the very rapid decrease in brightness in the former as compared with the very gradual falling-off in the latter object. The density near the stars is equal in the two cases, according to the curves, but an examination of Plate II reveals that there are a number of points near CD - 24° 12684 that are actually blacker than any near  $\rho$  Ophiuchi.

Two checks are available for testing independently the hypothesis of immersion. The comparison, in Table 1, of the spectral class, B<sub>3</sub>, and the apparent magnitude of CD - 24° 12684 with that of other B-type stars in the vicinity of the dark nebula indicates that it is about 3 mag. fainter than it should be. This weakening is what would be expected from the absorption of the material in front of the star. In a private communication Miss Cannon states that she

<sup>8</sup> Adams and Joy, *Mt. W. Contr.*, No. 244; *Ap. J.*, 56, 242, 1922.

<sup>9</sup> *Ap. J.*, 52, 14, 1920.

has re-examined the spectrum of this star and has found that the *He* lines are very well marked, precluding the possibility of a spectrum of later type. Consequently, any doubt about the dependability of the comparison in this table would come from the dispersion in absolute magnitude, although it must be admitted that a deviation of 3 mag. is rare. That some of the other stars in the table are not of high luminosity was ascertained by an examination of a num-

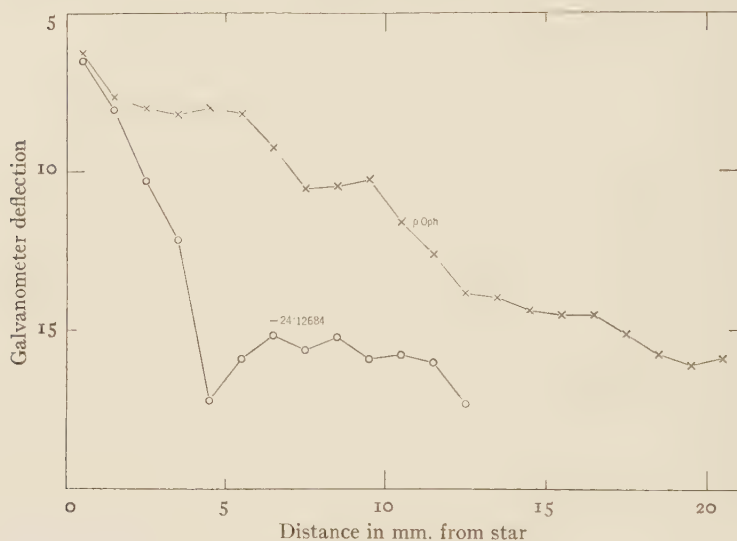


FIG. 8

ber of Yerkes spectrograms. No giant characteristics were found for  $\beta$ ,  $\sigma$ , and  $\nu$  Scorpii and for  $\rho$  Ophiuchi.

A second check is found in the color of the nebulosity. Two plates, one photographic and the other photovisual, which were taken with a Schmidt camera by Struve, Elvey, and Roach for their study of this region of nebulosity, were examined for color effects. It was found that, while the very bright filaments around  $\nu$  Scorpii were almost absent on the photovisual plate, the nebulosity in question was quite bright. Measurements with a recording microphotometer verified this observation and showed that there exists a difference in color index between these nebulae of about 0.2 mag. No correction was made for the color of the sky and the color of the diffuse illumi-

nation emanating from Antares, which is south of  $\alpha$  Scorpii and strongly illuminates the nebulosity around this latter star in long wave-lengths.<sup>10</sup> Corrections for these effects are uncertain and would merely increase the difference found; therefore none was applied. The redness of the nebula CD- $24^{\circ}12684$  may be explained by the fact that the source is shining through thick layers of nebulosity. This conclusion is verified in the most unexpected manner by the large color excess of  $-24^{\circ}12684$ , as found by Seares and Hubble. The small value of the color excess for  $\alpha$  Scorpii also agrees with our conclusions. That for  $\rho$  Ophiuchi is approximately one-half of that for  $-24^{\circ}12684$ . It is extremely desirable that these color excesses be redetermined with greater accuracy.

Assuming that  $\rho$  Ophiuchi is in front of the nebula, I have computed its distance from the front surface. The brightness at a point about  $30'$  north of  $\rho$  Ophiuchi is the same as that around Antares. Using the measures of Struve, Elvey, and Roach,<sup>11</sup> I find that this is  $0.7$  mag. fainter than the nebulosity immediately around  $\rho$  Ophiuchi. From the upper curve in Figure 4c this point is distant from the star by  $0.7x$ . Taking the parallax of the star as  $0''.01$ , we get, approximately,

$$x = 250,000 \text{ astronomical units.}$$

For an absolute magnitude of  $-0.6$  the apparent magnitude of the star at the nebula is  $-6.0$ . The brightness of the nebula, from the curve already used, is

$$11.2 - 6.0 - 2.5 \log \gamma.$$

<sup>10</sup> The strong red light coming from Antares and scattered by the nebula makes the total red light around  $\alpha$  Scorpii equal to that around CD- $24^{\circ}12684$ . This explains why no effect of reddening was detected by Struve, Elvey, and Roach in their original measurements on these plates. The measurements given in their Table 1 are substantially correct and do not conflict with the statements made in this paper. In fact, Struve states that he has visually found the nebula near  $\alpha$  Scorpii to be bluer than the one near  $-24^{\circ}12684$ .

<sup>11</sup> *Op. cit.*, Table 1. The references I make to these measures refer to their values corrected for the intensity of the sky.

From the measures of Struve, Elvey, and Roach this must be 1.2 mag. fainter than the sky, or about 5.7 mag. Equating and solving for  $\gamma$ , we find

$$\gamma = 0.6.$$

The measurements of CD - 24° 12684 have been fitted to the curve in Figure 6 for an optical depth of the source of 3 mag. Although no photometric standardization is available for any of the plates of Barnard, and although the measures were very rough, it is interesting to see what is obtained by assuming a linear relationship between intensity and galvanometer deflection. A fairly satisfactory fit was found when the distance of the star beyond the front surface was taken as 100,000 astronomical units, or 0.5 parsec. This result is interesting because it gives an approximate lower limit to the thickness of the nebula. In comparison with this, the diameter of the main central part of the dark nebula in Scorpius and Ophiuchus, not counting the arms, is about 5 parsec. Using this distance, we can compute another interesting quantity, the absorption coefficient, by dividing the absorption in the starlight by the distance just found; doing this, we get

$$k = 6 \text{ mag./parsec.}$$

This marked difference in the intensity distributions of the nebulosities we have discussed shows that Hubble's statistical method<sup>12</sup> cannot be applied to individual objects. For instance, the limiting diameter of the nebula associated with CD - 24° 12684 will behave differently, with varying exposure time, than will the diameter of a nebula for which the source is well outside of the medium.<sup>13</sup> Otherwise, the results derived in this paper are in good agreement with those obtained by Struve and Story from an application of Hubble's method.

## VII

The color excess of CD - 24° 12684, if interpreted as a result of Rayleigh scattering, would lead to a total photographic absorption

<sup>12</sup> *Mt. W. Contr.*, No. 250; *A p. J.*, **56**, 1922.

<sup>13</sup> However, diffuse reflection nebulae with the source inside are apparently rare, because of the large coefficient of absorption of nebular material, which causes both star and nebula to appear faint.



by small particles of the order of 1 mag. Since the observed absorption is at least 3 mag., we could attribute the remaining absorption of 2 mag. to large particles. It would be premature, however, to consider this result as final. Our observations have failed to give us a definite clue concerning the form of the phase function, and, while we have found no inconsistency with Lambert's function, it is true that other phase functions would have given equally satisfactory results. Our preference for large particles rests, as before, merely upon the evidence from the colors and from the degree of polarization. It is difficult to state the lower limit for the size of the particles consistent with these observations. Obviously, the particles cannot be very large since otherwise the mass of the nebula would be excessive. If we assume that the radius of an average particle

$$\rho = 10\lambda = 5 \times 10^{-4} \text{ cm},$$

we find from Russell's formula

$$\Delta m = 0.8 \frac{hd}{\rho g},$$

where  $d$  is the density of the medium and  $g$  is the specific weight of the particle, that the total mass of the nebula, for  $\Delta m = 6$  mag., is

$$\text{Area of nebula in } \text{cm}^2 \times hd \approx 10^3.$$

It is obvious that this mass is not excessive and that even particles having diameters of the order of 0.1 mm would give a mass that would not produce excessive velocities in the stars in its vicinity.

Professor B. Strömgren has pointed out to me that it would be interesting to investigate the possibility of the existence of small particles which absorb light but only scatter a very small amount of it. Such small particles may consist of metals which dissipate a large fraction of the incident energy in the Ohmic currents generated by the electric field of the radiation. By itself, the small amount of observed polarization in reflection nebulae does not preclude this possibility since these small particles would govern the properties of the scattered light only to a slight extent, the larger particles being the



most important sources of the nebular radiation. It is through their absorbing power, which may be selective, that such particles should reveal their presence. The coexistence of selective absorption in CD -  $24^{\circ}$  12684 and its nebula with a large value of the photographic absorption, on the one hand; the absence of strong selective effects in the reflected light of nebulae associated with unobscured stars, and the weakness of the polarization, on the other hand—all support the hypothesis that we are dealing with an agglomeration of large and small particles, both absorbing light but only the larger ones scattering it.

I wish to record my sincere appreciation to Professor Strömgren for his valuable criticism of the work, to Professor Kuiper for his various suggestions concerning the foregoing presentation, to Miss Annie J. Cannon for her kind re-examination of various spectra, and in particular to Professor Struve for his willing and generous co-operation during every phase of the work and for the inspiration which he has provided.

YERKES OBSERVATORY

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